PHOTON COUNTING SAMPLING OF PHASE SPACE

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The recently proposed scheme for direct sampling of the quantum phase space by photon counting is discussed within the Wigner function formalism.

First complete experimental characterisation of the quantum state of a single light mode was demonstrated by Smithey *et al.*[1] In their work, numerical tomographic algorithms were applied to reconstruct the Wigner function from homodyne statistics. Recently, a novel scheme for measuring the quantum state of light by photon counting has been proposed.[2] In contrast to tomographic techniques, the newly proposed method allows one to measure quasidistribution functions directly at a selected point of the phase space. In this contribution we show that the Wigner phase space representation provides a clear and intuitive interpretation of this scheme.

The principle of direct sampling of the quantum phase space by photon counting is to measure the photon statistics $\{p_n\}$ of the signal field $\hat{a}_S$, superposed on a probe field $\hat{a}_P$ by means of a beam splitter with the power transmission T. The measured photocount statistics is used to calculate the alternating series $\sum_{n=0}^{\infty}(-1)^n p_n$, which is given by the expectation value of the following normally ordered operator expressed in terms of the signal and the probe fields:

$$\hat{\Pi} = :\exp[-2(\sqrt{T}\hat{a}_S^\dagger - \sqrt{1-T}\hat{a}_P^\dagger)(\sqrt{T}\hat{a}_S - \sqrt{1-T}\hat{a}_P)]:. \tag{1}$$

We will interpret this observable using the quantum mechanical phase space formalism introduced by Eugene Wigner. For this purpose we will represent $\hat{\Pi}$ as an integral of the product of two quantities depending only on the signal or the probe mode operators:

$$\begin{aligned}\hat{\Pi} &= \frac{2}{\pi}\int \mathrm{d}^2\beta \ :\exp[-2(\sqrt{T}\beta^* - \hat{a}_P^\dagger)(\sqrt{T}\beta - \hat{a}_P)]: \\ &\quad \times :\exp[-2(\sqrt{1-T}\beta^* - \hat{a}_S^\dagger)(\sqrt{1-T}\beta - \hat{a}_S)]:. \end{aligned} \tag{2}$$

As the P mode is to be used as a probe for the S mode, we assume that both the modes are not correlated. Evaluating the quantum expectation value of the above integral representation yields the following formula:

$$\langle\hat{\Pi}\rangle = \frac{\pi}{2(1-T)}\int \mathrm{d}^2\beta \, W_S(\beta) W_P(\sqrt{T/(1-T)}\beta), \tag{3}$$

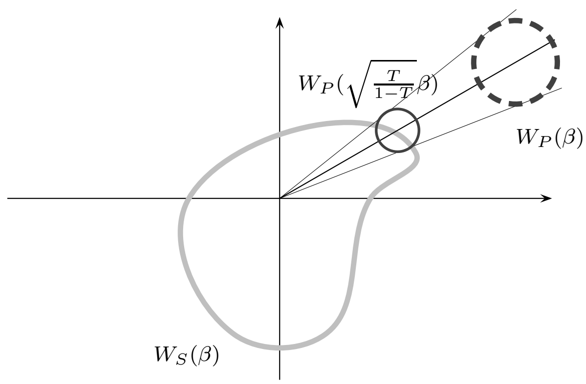

Figure 1: Phase space interpretation of the observable $\hat{\Pi}$ measured in the photon counting experiment.

which shows that the measured quantity is simply the phase space integral of the product of the signal and the probe Wigner functions $W_S(\beta)$ and $W_P(\beta)$. Thus, the quasidistribution function in the signal phase space is "sampled" by the probe Wigner function. Of course, the area of the "patch" of the probe Wigner function must be at least that imposed by the uncertainty principle. The essential advantage of the newly proposed scheme is that the parameterisation of the probe Wigner function in Eq. (3) is rescaled by the factor $\sqrt{T/(1-T)}$, which may take *any* positive value for $0 < T < 1$. When $T > 1/2$, the probe Wigner function becomes effectively "squeezed" in all directions, and consequently the resolution of the phase space measurement goes beyond the uncertainty relation limit. This feature is depicted schematically in Fig. 1. Let us stress that the rescaled Wigner function does not appear physically in the setup; it is only a tool for the phase space interpretation of the measured observable.

An important class of probe states which can be easily generated in a laboratory are coherent states $|\alpha\rangle$. In this particular case the outcome of the measurement can be expressed using an $s = -(1-T)/T$ ordered quasidistribution function of the signal mode:

$$\langle\hat{\Pi}\rangle = \frac{\pi}{2T} W_S\left(\sqrt{\frac{1-T}{T}}\alpha; -\frac{1-T}{T}\right). \tag{4}$$

In the limit $T \to 1$ the ordering parameter approaches zero, which corresponds to the direct measurement of the Wigner function of the signal field. The Wigner function is determined at a point defined by the amplitude and the phase of the probe coherent state. By changing these two parameters, the complete Wigner function can be scanned point–by–point, without using complex numerical algorithms.

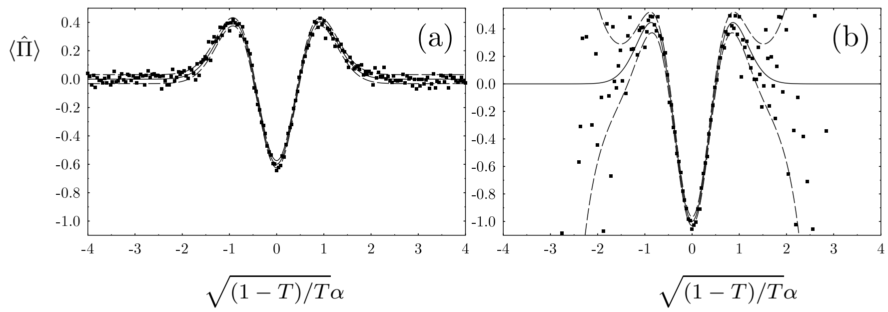

Figure 2: The one–photon Fock state quasidistribution function reconstructed from 1000 Monte Carlo events at each phase space point (a) without and (b) with compensation for the non–unit detector efficiency $\eta = 80\%$, in the limit $T \to 1$.

This result is easily understood within the Wigner function formalism. In the limit $T \to 1$ the rescaled Wigner function of the coherent probe approaches the shape of the delta function, and therefore the integral in Eq. (3) picks up the value of the signal Wigner function at a single phase space point. Additionally, the rescaling moves the probe Wigner function towards the phase space origin, which is reflected by the decreasing factor multiplying α in Eq. (4). Therefore, large intensity probe fields have to be used to scan the required region of the phase space.

In a realistic setup, there is a limitation on the resolution of the phase space sampling imposed by the non–unit quantum efficiency η of the photodetector. Furthermore, rigorous statistical analysis shows that an attempt of numerical compensation for this deleterious effect fails due to rapidly exploding statistical error. We illustrate this with Fig. 2, where Monte Carlo simulations of the photon counting experiment for the one–photon Fock state are superimposed on the exact expectation value with the analytically calculated statistical error. Detailed discussion of these issues can be found in Ref. 3.

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References

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